EXHIBIT A

to

AMENDMENT

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Saddle Coil for MRI

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Abstract: *The aim of this article is to apply the gradient design method to the RF Saddle coil to achieve the best homogenity of the magnetic field with the respect to good system matching. In the first part of the article a mathematical model is described. The Biot–Sarvat law is used to formulate the definitions for each coil element. Then the entire–coil analytical model is presented and the Taylor's magnetic field distribution inside the resonator is calculated. The gradients of the magnetic field are calculated and the optimization technique is shown.*

1. Introduction

The Saddle coil (also called resonator) is used in MRI/MRS (magnetic resonance imaging/scanning) applications in the position of gradient coils or RF probes. Gradient coils produce a linearly growing magnetic field along one axis and a homogenous magnetic field in the remaining ones, which gives us something like a coordinate system used for evaluating the measured data. To achieve the desired magnetic field inside the coil, the appropriate approximation of the magnetic field must be first evaluated. The type of approximation depends on the chosen coordinate system, for a cylindrical or spherical system the best choice is spherical approximation, for the linear coordinate system it is the Taylor's one. Because the gradient Saddle coil generates a homogenous x–y magnetic field and a linearly growing magnetic field along the z–axis, the Taylor's approximation is used. Applying the approximation in the center of the coil we can evaluate and optimize the gradients of the magnetic field by changing the parameters of the coil. RF Saddle coils should have a constant magnetic field in its entire area and serve as both the transmitter and the receiver for experiments. The major emphasis is put the good matching of the coil to the system instead on the achieving of a good homogenity of the magnetic field. The methods of the RF Saddle coil design are based on the inductance calculation and evaluation of the incident resonant frequency.

2. The method

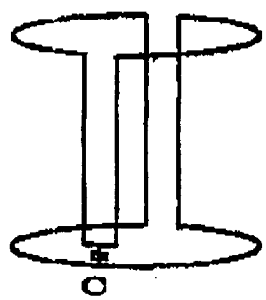

Fig. 1: The Saddle Coil. Capacitor C ensures proper resonance frequency

As it was said in the previous section, the evaluation of the gradient coil is based on the magnetic field calculations around the center of the coil. To use this method to describe and optimize the magnetic field of the RF resonator we must make some changes in the algorithm. The gradient design method is used to optimize the *linearity* of the magnetic inductance in the specified direction. We use the algorithm to optimize the *homogenity* of the magnetic field instead of achieving its maximum linearity.

The Saddle coils use the Taylor's approximation of the magnetic inductance inside the coil, because both Gradient and RF resonators use linear coordinate system to describe the field. The Taylor's approximation can be defined as:

$$f(x_0+h, y_0+k)=f(x_0,y_0)+\frac{\left(h\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)f_{x0,y0}}{1!}+\ldots+\frac{\left(h\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^n f_{x0,y0}}{n!}+R_{n+1} \quad . \quad (1)$$

where $f(x_0,y_0)$ is the approximated function, n is the order of the approximation, and R_{n+1} is the Lagrange's rest of the function. The function $f(x,y)$ must have a total differential to $(n+1)th$ order. To get the desired precision of the approximation of the magnetic inductance in the center of the coil we must choose the order of the function. Obviously the approximation of the fifth order is used, but for the purposes of this article we will use the third order because of the big complexity of the expressions. The 3rd order Taylor's approximation defined for the three-dimensional function of the magnetic inductance can be expressed as:

$$B_x(0,0,0)=B_x(0,0,0)+\frac{1}{1!}\left[\frac{\partial B_x}{\partial x}x+\frac{\partial B_x}{\partial y}y+\frac{\partial B_x}{\partial z}z\right]^1+\ldots+\frac{1}{2!}\left[\frac{\partial B_x}{\partial x}x+\frac{\partial B_x}{\partial y}y+\frac{\partial B_x}{\partial z}z\right]^3=$$
$$=\frac{\partial B_x}{\partial x}x+\frac{\partial B_x}{\partial y}y+\frac{\partial B_x}{\partial z}z+\frac{\partial^2 B_x}{\partial x^2}x+2xy\frac{\partial^2 B_x}{\partial x\partial y}+\ldots \tag{2}$$

Because it is the x direction, what interests us (i.e. a major homogenity must be well defined in the x direction), the approximation of the B_x is made. As can be seen, the 3rd order of the approximation gives us fairly complicated formula, which can be divided into the groups with the specified order. To obtain the result we must evaluate each of the derivations in the equation (2) with the respect to their order. The lower orders give us the major contribution to the inhomogenity of the field, so we must take care about them.

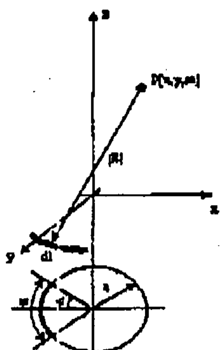

Fig. 2: Values used to calculate the magnetic inductance

To calculate the approximation (2) inside the entire coil's space we must know a magnetic field produced by a single piece of the wire, having radius of a. The information about the magnetic field gives us the Biot–Sarvat law:

$$B_0=\frac{\mu I}{4\pi}\int\frac{d\vec{l}\times R}{R^3}, \tag{3}$$

where I is the current flowing through the wire, R is the direct distance from the point of view P, specified by three coordinates (x,y,m). u is the permeability of the background. For the wire depicted at fig. 2 we can write:

$$d\vec{l}=a\,d\varphi\quad \Phi=-x\sin\varphi+y\cos\varphi+0z, \tag{4}$$

$$R=(x-a\cos\varphi)x+(y-a\sin\varphi)y+mz, \tag{5}$$

where the vectors R, and dl are also composed of the three space components. To obtain the result we substitute the eq. (4) and (5) into (3):

$$B_0=\frac{\mu_0 I}{4\pi}\int_{-\alpha}^{\alpha}\frac{\begin{vmatrix} x & y & z\\ -\sin\varphi & \cos\varphi & 0\\ x-a\cos\varphi & y-a\sin\varphi & m\end{vmatrix}}{\sqrt{(x-a\cos\varphi)^2+(y-a\sin\varphi)^2+(z+m)^2}} \tag{6}$$

The results for rounded wire and the wire parallel to z axis are shown in Tab. 1. All the variables can be understood from the Fig. 2. The formulas from Tab. 1 are used to evaluate the Taylor's approximation of the magnetic field. There can be seen, that the equations of the magnetic inductance are complex, and in fact, there exist no analytic solutions for Bx, By and Bz. This disadvantage can be eliminated by the eq. (2), where we can see, that there are in the expression the derivatives only, so to get the solution, the derivatives of the integral equations from Tab. 1 must be calculated. The calculations can be made for the x axis only, because the RF probe generates the magnetic field mainly in this direction. Let's say, that the derivatives of the equations shown in the Tab. 1 are proportional

to the integral solution. In this case the derivative of the solution is given by integrating the derivative of the integrant.

Tab. 1: The magnetic inductance for two types of wire used in the Saddle coil

B	Wire with radius a	Wire parallel to z
x	$\frac{\mu_0 Ia}{4\pi}\int\frac{\cos\varphi(z+m)}{[q-2a(x\cos\varphi+y\sin\varphi)]^{3/2}}$	$\frac{\mu_0 Ia}{4\pi}\int\frac{(y-a\sin\psi)dz}{[q-2a(x\cos\varphi+y\sin\varphi)]^{3/2}}$
y	$\frac{\mu_0 Ia}{4\pi}\int\frac{\sin\varphi(z+m)}{[q-2a(x\cos\varphi+y\sin\varphi)]^{3/2}}$	$\frac{\mu_0 Ia}{4\pi}\int\frac{(x-a\cos\psi)dz}{[q-2a(x\cos\varphi+y\sin\varphi)]^{3/2}}$
z	$\frac{\mu_0 Ia}{4\pi}\int\frac{a-x\cos\varphi-y\sin\varphi}{[q-2a(x\cos\varphi+y\sin\varphi)]^{3/2}}$	0
where $q=x^2+y^2+a^2+(z+m)^2$		

For the case of B_x magnetic inductance, and $\partial B_x/\partial x$ derivative, which gives us the most important magnetic field inhomogenity we can write:

$$\frac{\partial B_x}{\partial x}=\frac{\mu_0 I}{4\pi}\int\frac{\partial\left[\frac{\cos\varphi(z+m)}{[q-2a(x\cos\varphi+y\sin\varphi)]^{3/2}}\right]}{\partial x}=\frac{\mu_0 I}{4\pi}\int\frac{-3}{2}\frac{\cos(\varphi)(z+m)}{[q-2a(x\cos\varphi+y\sin\varphi)]^{(5/2)}} \tag{7}$$

The Taylor's approximation is formed around the center of the coil, where the coordinates x,y,z can be replaced by (0,0,0). In this case we can write:

$$\frac{\partial B_x}{\partial x}=\frac{3\mu_0 I}{4\pi}\int\frac{\cos\varphi^2 ma}{(a^2+m^2)^{(5/2)}} \tag{8}$$

The solution of this equation leads to the simple formula, giving us the amount of the field's inhomogenity referred to the center of the coil. The integrated form of the equation (8) can be written as:

$$\frac{\partial B_x}{\partial x}=\frac{3}{4}\frac{I\mu\beta(\cos\psi\sin\psi+\psi)}{a^2\pi(1+\beta^2)^2\sqrt{a^2(1+\beta^2)}}, \tag{9}$$

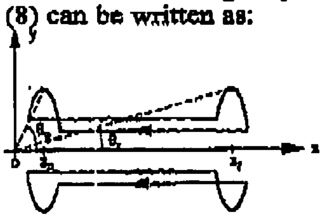

Fig 3: The saddle coil wire system

where ψ is the angle of the curved strip and $\beta=m/a$ is simple ratio between the length and diameter of the coil. This is the fundamental result of describing the magnetic field gradients produced by each element of the coil. The result is only partial and all the important values of the differentials up to the specified order expressed in eq. (2) must be evaluated to get the entire Taylor's approximation. The approximation describes only the behavior of the one element of the coil. To get the approximation for the entire coil's magnetic inductance we must sum all the contributions from each of the element. The number of the elements can be seen from Fig. 3. The curved wires can be solves by the expressions from the second column of the Tab. 1, the wires parallel to z from the third. Some of the derivations vanish due to the symetricity of the coil, some are doubled. In fact, due to reciprocity of sum of the approximations for Bx, we need not to sum the equation (2) to form the coil's wiring system, instead of that we can sum only the appropriate derivations like eq. (9), which is more useful result for us. The optimum parameters of the coil are reached when the most important derivations vanish due to good

dimensions and the currents directions in the coil. The derivations show us the amount of the change of the magnetic field in the given area. When the value vanish, the intensity of the magnetic field is meant to be constant in the specified direction and in the order specified by the order of the derivation. As an example here we can show, that the derivation shown in eq. (9) vanish when the conditions specified in the eq. (10,11,12) are reached.

$$\beta=\frac{m}{a}=0 \tag{10}$$

$$I=0 \tag{11}$$

$$\cos\psi\sin\psi+\psi=0 \tag{12}$$

The first two results aren't useful, the third has no solution in the real space. This means, that the coil's first derivative in this configuration cannot vanish, because the system cannot achieve the condition. The only possibility to get rid of the derivation is change the type of the coil, respectively in our case for example make the coil consisting of multiple turns of wire, what makes us the difference between gradient and RF coils. The gradient coils are optimized in the z direction and the magnetic field produced by the parts of the coil parallel to z axis vanish due to the opposite current directions in the coil (the coil depicted in Fig. 3 is also called Helmholtz coil, having the same current directions and producing the homogenous magnetic field in the x direction. There exists a similar coil called Gradient–Maxwell, which produces linearly growing magnetic field in the z direction and has the opposite current directions.). While the most important gradient in the z direction is linearized, the other gradients can be removed by changing the number of turns of the coil or by the good combination of more Saddle coils together, eventually by good combination of currents in the coils. The RF coils are optimized in the x direction, where the parts parallel to the z axis have unneglecting influence on the specified magnetic field. Instead of linearizing the important gradients, they must vanish to ensure the magnetic field homogenity. Moreover the RF probes used in biological experiments cannot be in the majority made of multiple turns, because the coils operate at high frequencies, where the multiple turns unfavorable affect the coils resonance modes.

Although the main first derivation can be in our case minimized only, there can be optimized some of the other derivatives. The optimization gives us the information, that the coil should have the diameter equal to it's length and the optimum angle of the strip is 120 degrees.

3. Conclusion

In this article the modification of the gradient optimization method was presented. The method was applied to optimize the homogenity of the RF Saddle coil. Although this method works well for the gradient coils, it can be seen, that the method is only partially suitable for the RF Saddle coils. The main reason is that the difference between the types of the magnetic fields baffles annulling of the most important derivative in the x direction. The derivative can only be minimized, which leads to the diameter of the coil equal to its length and the 120 degrees of the optimum angle for the curved strips.

Acknowledgement

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